Politician Preferences, Law-Abiding Lobbyists and Caps on Political Lobbying

March 19, 2009

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Abstract

The effect of a contribution cap is analyzed in a political lobbying game where the politician has a preference for the policy position of one of the lobbyists. In contrast to the previous literature where the politician has no preference over policy alternatives, we find that a more restrictive binding cap always reduces expected aggregate contributions. However the initial imposition of a cap will increase contributions if and only if the politician mildly favors the policy of the low valuation lobbyist. The introduction of policy preferences permits the analysis of the effect of a cap on the influence of monied interests. In equilibrium a more restrictive cap makes it more likely that the politician enacts the policy alternative he would have enacted in the absence of lobbying, even in cases where the expected aggregate contributions increase.

Keywords: All-pay auction, campaign finance reform, explicit ceiling.
JEL: D72, C72

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> "Because it costs so much to run for office, interests with big money to contribute to candidates or spend on ad campaigns are able to get special access in Congress."
>
> Senator Russ Feingold (D-WI)

> "Americans believe that political representation is measured on a sliding scale. The more you give the more effectively you can petition your government."
>
> Senator John McCain (R-AZ)[1]

1. Introduction

The concept of representative democracy is founded on the proposition that the actions of elected representatives in some sense reflect the will of the people. Either the public votes for people whose views reflect their own, or the desire to be reelected leads politicians to try to act as though their views reflected the public's. In either case it is likely that an elected politician has preferences over policy alternatives.

There is concern that the need to raise money to finance election campaigns is diluting this fundamental premise of representative democracy. In 2008 the average cost of a successful campaign for the House of Representatives was $1.3 million, which represents a real increase of 53% in a decade. Over the same period the average cost of a winning Senate campaign increased by 21% in real terms to $6.5 million.[2] The need to raise funds takes time away from other duties and raises the concern that legislative outcomes may be driven by money.[3]

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[1] Quoted on the Senators' web sites, March 2008.

[2] For summary statistics see the web sites of the Campaign Finance Institute and the Center for Responsive Politics, www.cfinst.org and www.opensecrets.org respectively.

[3] It is well documented that larger contributors are more likely to gain access to legislators and that they lobby members with positions of power in congressional committees more heavily (Hall and Wayman (1990), Langbein (1986) Tripathi et al (2002) and Wright (1990)). There is extensive literature documenting that institutional contributors appear to be acting as rational investors (see Ansolabehere and Snyder (1999), Grier and Munger (1991), Grier et al (1994), Hart (2001), Kroszner and Stratmann (1998, 2000), Lott (2000), Milyo (1997), Pittman (1988), Romer and Snyder (1994), Snyder (1990, 1992, 1993), and Zardkoohi (1998). Contributions may also be viewed as a way of influencing elections or as pure consumption. These motivations are not mutually exclusive.

In the U.S. there have been numerous attempts to regulate campaign financing by imposing caps on political contributions.[4] The current Federal regulation on campaign financing is the Bipartisan Campaign Reform Act of 2002, also known as the McCain-Feingold Bill. The act limits an individual's contributions to a candidate to a maximum of $2300 per election and to a political action committee to a maximum of $5000 with built-in increases for inflation. However it is a complicated piece of legislation which provides various avenues for contributors to direct funds in support of a candidate. The current effective legal limit on an individual's total contributions is $70,100 in any two-year period.[5]

Caps on political contributions are put in place with the desire to reduce the influence of special interest groups by lowering the total special interest group money in politics.[6] Natural intuition suggests that contribution caps would result in decreased aggregate contributions. However Che and Gale (1998), henceforth CG, challenges this intuition in an all-pay auction setting where lobbyists have different valuations of a political prize. CG shows that a more restrictive cap can level the playing field inducing higher aggregate contributions from lobbyists.[7] In CG the politician has

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[4]A number of other counties also have contribution limits. Examples include France, India, Israel, Italy, Japan, Mexico, Russia, Spain, Taiwan and Turkey. See www.aceproject.org.

[5]For the contributions limit chart see http://www.fec.gov/pages/brochures/contriblimits.shtml. See the Federal Election Commission's website www.fec.com for details. For state-level offices individual states are in charge of their own campaign finance regulations. All states except for Illinois, New Mexico, Oregon, Utah and Virginia have contribution limits. Details on various state level contribution limits are provided by the National Conference of State Legislatures, www.ncsl.org.

[6]Interest groups may be contributing to support politicians who share their values, rather than to buy their votes. In order to establish clear causality between money and voting behavior, Stratmann (2002) examines repeated votes on the same piece of legislation: the repeal of provisions of the 1933 Glass-Steagall Act. The act prohibited bank holding companies from owning other financial services companies. The repeal was rejected by the House in 1991, and it then passed in 1998. It was strongly favored by banking interests but also strongly opposed by insurance and securities interests. Stratmann finds that an extra $10,000 in contributions was associated with an 8% increase in the probability of a House member voting to repeal the prohibition.

[7]Drazen, Limão and Stratmann (2007) find a related result in a very different framework. In an incomplete information environment Gavious, Moldovanu and Sela (2002) find that expected spending can go up when the cap is more restrictive. Amegashie (2003) analyzes caps in all-pay auctions when a committee awards the prize.

no preference over the policy alternatives supported by the lobbyists. This paper extends CG by allowing the politician to have a preference for the policy position of one of the lobbyists contesting for the political prize.[8]

Kaplan and Wettstein (2006) and Gale and Che (2006) analyze caps when lobbyists may be willing to break the law and possibly contribute above the legally set limit. Here we continue to maintain the CG assumption that lobbyists are law abiding and do not attempt to circumvent the law as written. Hence we analyze the effect of a contribution cap in the baseline case where the law operates as intended.[9]

In contrast to CG, we find that a more restrictive binding cap always decreases expected aggregate contributions no matter how mild the policy preference may be. The lobbyist with the preferred policy position does not need to match his rival's contribution in order to win. This implies that the effect of the cap is qualitatively different from the effect of the cap when the politician is indifferent between policy alternatives. In CG both lobbyists are constrained by the cap: Given their rival's strategy they would each like to exceed the limit if it were possible to do so. However, when the politician has a policy preference, the cap effectively constrains the less-preferred lobbyist, but not the preferred lobbyist. The favored lobbyist never needs to contribute by the amount of the cap in order to guarantee victory since the unfavored lobbyist cannot contribute more than the cap. Hence the cap always helps the preferred lobbyist. A more restrictive binding cap tilts the playing field in favor of the preferred lobbyist, reducing the aggressiveness of his rival. This leads to decreased expected contributions overall.

If the politician mildly prefers the policy position of the low-valuation lobbyist, the main message of CG that a contribution limit may increase expected total contributions survives at the

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[8]There is extensive empirical evidence that the policy position of the politician is an important determinant of politician behavior. Of the 36 empirical papers which study ideology or party affiliation surveyed in Ansolabehere *et al* (2003), all but one find policy position significant for predicting congressional roll-call votes.

[9]See Pastine and Pastine (2008) for the case where the politician has policy preferences and lobbyists circumvent the cap.

point where the cap just becomes binding. In this case the preference of the politician is not too strong, so without a binding cap the lobbyist with the higher valuation of the political prize is in an advantageous position. Introduction of a binding cap switches the advantage to the favored lobbyist (the low-valuation lobbyist). This fosters more aggressive bidding by the low-valuation lobbyist and results in higher expected aggregate contributions. Hence a politician who is concerned with raising money may support a barely binding cap over no cap.

The introduction of policy preferences permits the analysis of the effect of a cap on the influence of monied interests. The literature often cites the level of total political contributions as a measure of the degree of influence of money in politics. We suggest a different measure that captures the concern that money may be driving policy choices: the equilibrium probability that the politician does not enact policy that he would have enacted in the absence of lobbying. The choice of measure matters. With policy preferences the imposition of a cap may lead to increased aggregate contributions while making it more likely that the politician goes with his conscience, reducing the influence of lobbying effort. We find that a more restrictive cap always makes it more likely that the politician enacts his favored policy. Furthermore lobbying activity will be observed on fewer policy issues.

However our theoretical findings imply that empirical evaluation of the effect of a cap is nontrivial. We find that even when the cap is significantly restricting donations, in equilibrium very few contributions will be at the limit. Hence the standard practice of looking at the proportion of donations at the maximum permitted amount, as for example in Ansolabehere *et al* (2003), may fail to identify binding caps. We also show that contribution caps may redistribute political contributions from Senators and politicians from large or urban districts to Representatives and politicians from smaller or rural districts.

To the best of our knowledge this is the first paper to characterize the equilibrium of a preferential treatment all-pay auction with a cap. We first analyze the equilibrium of the lobbying game without a cap. We adapt Konrad's (2002) all-pay auction with additive preferential treatment to allow bidders to have different valuations of the prize. We then examine the effect of a cap on contributions. We conclude with a short discussion of a possible extension to help study campaign finance regulations in the European context where the cap is on expenditures rather than on contributions.

2. The Model

Two risk-neutral lobbyists compete for a political prize. The prize arises due to a policy choice of a politician who holds a political post. The prize may be a vote on impending legislation but may also be more subtle, such as attaching a rider to an upcoming bill creating a regulatory loophole, or pushing a particular wording in a committee. The value of the political prize to lobbyist 1 is denoted by v_1, and the value of the prize to lobbyist 2 is v_2, $v_1>v_2>0$. The lobbyists make simultaneous contributions (bids), b_1 and b_2, to the politician in power. The contributions are not returned to the lobbyist whose efforts fail. Since the contributions are sunk both for the winner and the looser, this political lobbying game is an all-pay auction.[10] If bidder 1 (lobbyist 1) wins the prize, his payoff is v_1-b_1, if his rival wins bidder 1's payoff is -b_1. Bidder 2's (lobbyist 2's) payoffs are constructed in the same manner.

In this paper we allow the politician to have a preference over the policy alternatives supported by the two lobbyists. The politician's preference may be ideologically based or it may be induced from the preferences of constituents who will be voting in the future. The interest groups lobby the politician and the politician awards the political prize based on the contributions and his preference. The lobbyist with the preferred policy position has an advantage since he can win the prize with a smaller contribution than his rival's. The degree of the advantage depends on the intensity of the preference of the politician.

[10]The all-pay auction without a cap has been analyzed by Hillman and Riley (1989), and Baye *et al.* (1993, 1996). See Yildirim (2005) for a contest where players have the flexibility to add to their previous efforts and see Kaplan *et al.* (2002) for a model where the size of the reward is a function of the bid. Prat (2002) has a model with multiple lobbyists contributing to competing politicians.

The intensity of the preference for the policy position of lobbyist 2 is put into monetary terms, denoted $\gamma \in (-\infty, \infty)$. For example $|\gamma|$ could represent the expected future campaign costs required to offset the effect of taking a policy position that is unpopular in the politician's district. If the politician favors lobbyist 2's position, $\gamma>0$. If the politician favors lobbyist 1's position, $\gamma<0$. It will be possible to write the proofs much more concisely if we define f as the bidder whose policy is favored by the politician and u as the bidder with the unfavored policy. If $\gamma \geq 0$ then f=2 and u=1, while if $\gamma<0$ then f=1 and u=2. It will be assumed that the politician awards the prize to lobbyist 1 if $b_1>b_2+\gamma$, and to lobbyist 2 if $b_1<b_2+\gamma$. In case of a tie, $b_1=b_2+\gamma$, each contestant has an even chance of winning the prize. CG is a special case of our framework where the politician does not have a policy preference, γ=0. The rules of the game, the valuations of the lobbyists and the preference of the politician are common knowledge.

Simple backward induction in the one-shot game that will be analyzed here would have the politician taking his preferred action regardless of bids since all contributions are sunk. Hence there would be no contributions. Thus implicitly we assume that this one-shot game is embedded in a repeated setting so that the politician has an incentive to reward high contributions in order to keep them coming in the future. However, as long as contributions, preferences and actions are common knowledge among lobbyists, the same lobbyists do not necessarily need to be involved in repeated contests.

3. Equilibrium without a Cap

If the politician's preference is too strong, either $\gamma \geq v_1$ or $\gamma \leq -v_2$, the unique equilibrium is in pure strategies where neither lobbyist contributes. The preferred lobbyist can bid zero and still win the prize since it would never be optimal for his rival to contribute more than his valuation. We study all nontrivial cases where the politician has a policy preference $\gamma \in (-v_2, v_1)$. Equilibrium of this contribution game does not exist in pure strategies. The best response to a bid b' of the favored bidder is either to outbid the rival by $|\gamma|$ or to drop out of the race, so b' would not be optimal.

Lemma 1 below describes the equilibrium. This lemma extends Konrad (2002) to allow the value of the prize to differ between bidders. In Konrad (2002) the bidder with the head-start advantage (the lobbyist with the favored policy in our framework) always has a positive expected value from the contest and the bidder without the head-start advantage has an expected value of zero. However in our framework where bidders have different valuations of the prize, this is not always the case. When the politician mildly prefers the policy position of the low-valuation lobbyist, the preferential treatment is not strong enough to overwhelm the advantage lobbyist 1 has due to his high-valuation. This implies that we need to study the equilibrium in two separate cases.

Lemma 1: *Without a contribution cap, there is no equilibrium in pure strategies if* $\gamma\in(-v_2,v_1)$. *The equilibrium in mixed strategies is characterized by unique cumulative density functions* $F_f(b)$ *and* $F_u(b)$ *for the favored lobbyist's and the unfavored lobbyist's contributions, respectively.*

(i) *If the politician favors the policy position of the high-valuation lobbyist* $\gamma\in(-v_2,0)$ *or if the politician "strongly favors" the policy position of the low-valuation lobbyist* $\gamma\in(v_1-v_2,v_1)$, *the unique equilibrium cumulative density functions are given by*

$$F_f(b)=\begin{cases}\dfrac{b+|\gamma|}{v_u} & \text{for } b\in[0,v_u-|\gamma|]\\ 1 & \text{for } b>v_u-|\gamma|\end{cases}$$

and

$$F_u(b)=\begin{cases}\dfrac{v_f-v_u+|\gamma|}{v_f} & \text{for } b\in[0,|\gamma|]\\ \dfrac{v_f-v_u+b}{v_f} & \text{for } b\in(|\gamma|,v_u]\\ 1 & \text{for } b>v_u\end{cases}$$

(ii) *If the politician "mildly favors" the policy position of the low-valuation lobbyist* $\gamma\in(0,v_1\text{-}v_2]$*, the unique equilibrium cumulative density functions are given by*

$$F_f(b)=\begin{cases}\dfrac{v_u-v_f+b}{v_u} & for \quad b\in[0,v_f]\\ 1 & for \quad b>v_f\end{cases}$$

and

$$F_u(b)=\begin{cases}0 & for \quad b\in[0,|\gamma|]\\ \dfrac{b-|\gamma|}{v_f} & for \quad b\in(|\gamma|,v_f+|\gamma|]\\ 1 & for \quad b>v_f+|\gamma|\end{cases}$$

The proof of Lemma 1 is in Appendix A.

It is straightforward to derive the expected contributions of individual bidders and the probabilities of winning from the equilibrium distribution functions.

(*i*) $\gamma\in(-v_2,0)\cup\gamma\in(v_1\text{-}v_2,v_1)$. On $b\in(|\gamma|,v_u]$ the p.d.f. of the bids of bidder u is $f_u(b)=1/v_f$. The expected contribution of bidder u is given by

$$E(b_u)=\int_{b_u=|\gamma|}^{v_u}x f_u(x)dx=\frac{v_u^2-\gamma^2}{2v_f}$$

On $b\in(0,v_u\text{-}|\gamma|]$ the p.d.f. of the bids of bidder f is $f_f(b)=1/v_u$. The expected contribution of bidder f is given by

$$E(b_f)=\int_{b_f=0}^{v_u-|\gamma|}x f_f(x)dx=\frac{(v_u-|\gamma|)^2}{2v_u}$$

When bidder u contributes by an amount b, he wins the contest if and only if bidder f contributes less than b-$|\gamma|$. Hence the probability that bidder u wins the contest is given by

$$prob_u=\int_{b=|\gamma|}^{v_u}F_f(x-|\gamma|)f_u(x)dx=\int_{b=|\gamma|}^{v_u}\frac{x}{v_u v_f}dx=(v_u^2-\gamma^2)/2v_u v_f$$

(*ii*) $\gamma \in (0, v_1 - v_2]$. f=2 and u=1. On $b \in (|\gamma|, v_f + |\gamma|]$ the p.d.f. of the bids of bidder u is $f_u(b) = 1/v_f$. The expected contribution of bidder u is given by

$$E(b_u) = \int_{b=\gamma^+}^{v_f+\gamma} x f_u(x)dx = \frac{v_f + 2\gamma}{2}$$

On $b \in (0, v_f]$ the p.d.f. of the bids of bidder f is $f_f(b) = 1/v_u$. The expected contribution of bidder f is given by

$$E(b_f) = \int_{b_f=0}^{v_f} x f_f(x)dx = \frac{v_f^2}{2v_u}$$

The probability that bidder u wins the contest is given by

$$prob_u = \int_{b=\gamma^+}^{v_f+\gamma} F_f(x - |\gamma|) f_u(x)dx = 1 - v_f / 2v_u$$

When the politician mildly favors the policy of the low-valuation bidder, an increase in the intensity of the preference parameter has no effect on the equilibrium probabilities of winning. In this range, the preference of the politician is simply offset by the greater effort of lobbyist 1 (bidder *u*) while the expected effort from lobbyist 2 (bidder *f*) remains unchanged.[11]

When the politician is indifferent between policy alternatives, lobbyist 2 has a disadvantage in the game due to his low valuation of the prize. When the politician has a policy preference, the expected aggregate contributions has a maximum at the preference parameter $\gamma = v_1 - v_2$. At this level of γ the disadvantage of lobbyist 2 due to his lower valuation is just offset and the playing field is leveled (the expected value of the contest to both of the lobbyists is equal to zero). On a level playing field both lobbyists are most aggressive.

[11]This result is different from the affirmative action paper of Fu (2006) where preferential treatment is modeled as a multiplicative weight. A multiplicative preferential treatment rule augments the bid of the favored bidder by a fixed percentage which gives that bidder an additional incentive to increase his effort. Pastine and Pastine (2009) explores the implications of this difference for affirmative action policy.

4. Equilibrium with a Cap

Denote m as the level of the contribution cap. The lobbyists are assumed to be law abiding. Hence neither bidder contributes more than m. A cap restricting contributions to $|\gamma|$ or less would result in the unfavored lobbyist being unable to compete at all. Hence if the cap is too restrictive it completely suppresses all contributions. What follows discusses the nontrivial case where the cap permits contributions greater than the preference parameter, $m > |\gamma|$.

First define some terminology. A "*binding cap*" is a cap which is lower than the maximum of the upper bounds of the no-cap equilibrium bid supports. The supports of the equilibrium bids are established in Lemma 1. (*i*) If the politician favors the high-valuation lobbyist or if the politician strongly favors the low-valuation lobbyist, in the absence of a cap the favored lobbyist mixes in the range $[0,v_u-|\gamma|]$ and the unfavored lobbyist mixes in the range $\{0\}\cup(|\gamma|,v_u]$. Hence a cap $m<v_u$ is binding. (*ii*) If the politician mildly favors the low-valuation lobbyist, a cap $m<v_f+|\gamma|$ is binding. A cap that is ε less than the maximum of the upper bounds of the supports of the no-cap equilibrium bids is a "*barely binding*" cap. A "*more restrictive cap*" refers to a smaller m when the cap is binding.

Lemma 2 below describes the equilibrium with a cap when the politician has a policy preference. As long as the cap does not suppress all contributions, $m>|\gamma|$, there is no pure-strategy Nash equilibrium. This result is in contrast to CG. When the politician does not have a policy preference (γ=0) the nature of the equilibrium changes from a mixed-strategy equilibrium to a pure-strategy equilibrium when a very restrictive cap is introduced $(m<v_2/2)$ and both bidders contribute by the amount of the cap. When the politician has a policy preference, the favored bidder's optimal response to a bid b' is either to bid slightly higher than $b'-|\gamma|$ or to drop out of the contest altogether, so b' would not be optimal for the unfavored lobbyist. The unique equilibrium is in mixed strategies.

Lemma 2: *With a binding contribution cap and m>|γ|, there is no pure-strategy equilibrium if* $\gamma \in (-v_2, 0) \cup (0, v_1)$. *The equilibrium is characterized by unique cumulative density functions* $F_f(b)$ *and* $F_u(b)$ *for the favored lobbyist's and the unfavored lobbyist's contributions, respectively.*

$$F_f(b) = \begin{cases} \dfrac{b+|\gamma|}{v_u} & for \quad b \in [0, m-|\gamma|] \\ 1 & for \quad b > m-|\gamma| \end{cases}$$

and

$$F_u(b) = \begin{cases} \dfrac{v_f - m + |\gamma|}{v_f} & for \quad b \in [0, |\gamma|] \\ \dfrac{v_f - m + b}{v_f} & for \quad b \in (|\gamma|, m] \\ 1 & for \quad b > m \end{cases}$$

The proof of Lemma 2 is in Appendix A. The equilibrium distribution functions of bidder *u* and bidder *f* are graphed in Figure 1.

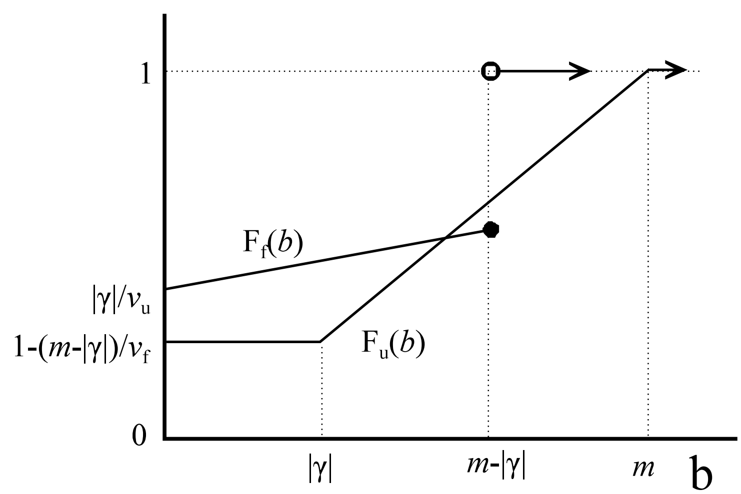

Figure 1: Equilibrium bids with a binding contribution cap. Bidder *f*'s policy is favored by the politician.

An important feature of the equilibrium is that the favored lobbyist never bids up to the cap. Since the unfavored lobbyist cannot contribute more than the cap, the favored lobbyist always has the option of winning for sure with a contribution just above m-$|\gamma|$. Also note that in equilibrium the unfavored lobbyist has a negligible probability of contributing the maximum amount. This implies that it will be difficult to establish empirically whether an existing contribution cap is binding or not. Natural intuition would suggest that if the cap were binding there would be large numbers of lobbyists who contribute the maximum permissible amount. Ansolabehere *et al* (2003) argues that the constraint on political contributions is not binding since only 4% of PAC contributions to House and Senate candidates are at or near the legal limit. However, Lemma 2 shows that in equilibrium neither lobbyist has a probability mass at the contribution cap. The favored lobbyist does have a probability mass at the maximum permissible amount less the politician's policy preference. However one would not expect to see this mass point in actual data since in practice different policy issues are likely to induce different intensities of preferences. Instead one would expect to see the distribution of contributions peaking below the cap, reflecting the underlying distribution of the preference parameter over different policy issues.

In equilibrium it is possible that the unfavored lobbyist contributes more than the favored lobbyist but not by enough to overcome the politician's preference. Consequently, in an empirical study the evidence of the effect of money on legislative action may appear to be weak. Indeed in their survey Ansolabehere *et al* (2003) find that empirical evidence on the effect of PAC contributions on roll-call votes is mixed.[12] Furthermore given that the preference of the politician would vary over policy issues, the model is consistent with the fact that the evidence appears to be strong in some policy areas but not in others.

[12]They survey 34 empirical papers and find that evidence on the effect of PAC contributions on roll-call votes is strong in some policy areas but not in others. For instance, on issues relating to trade there is weak evidence of the effect of PAC contributions on votes, but on issues relating to labor the evidence is very strong.

When the politician has a preference over policy alternatives, however mild the preference may be, the equilibrium predictions are different from the case where $\gamma=0$. Lobbyist 1 has an advantage in the contest due to his higher valuation of the political prize. CG shows that with $\gamma=0$ a very restrictive cap levels the playing field. This induces the low-valuation lobbyist to become more aggressive and both lobbyists contribute by the maximum legal limit. The expected aggregate contributions go up. However, when the politician has a policy preference a more restrictive binding cap always tilts the playing field in favor of the preferred lobbyist irrespective of the identity of the low-valuation lobbyist and expected aggregate contributions go down.

Proposition 1: *For all $\gamma\neq 0$, making a binding cap more restrictive always reduces expected aggregate contributions.*

The calculation of expected aggregate contributions and the proof of Proposition 1 are in Appendix B. Figure 2 gives the expected aggregate contribution as a function of m for possible ranges of γ, as defined in Lemma 1.

The lobbyist with the unfavored policy is constrained by the contribution cap. But the favored lobbyist is not effectively constrained since he never needs to contribute above m-$|\gamma|$ to guarantee victory. This advantage allows the favored bidder to capture a strictly positive expected value from the contest equal to v_f -m+$|\gamma|$. Hence if the cap becomes more restrictive the unfavored lobbyist becomes more constrained which is to the advantage of the favored lobbyist. As the cap gets more restrictive, the playing field is tilted more in favor of the preferred lobbyist. This decreases in the overall aggressiveness of the unfavored lobbyist, which in turn induces less aggressive bidding from the preferred lobbyist, leading to decreased expected aggregate contributions. So, the natural intuition put forward by proponents of campaign finance reform is indeed correct when the politician has a preference over policy alternatives. Further tightening an existing binding contribution cap always reduces expected aggregate contributions in equilibrium.

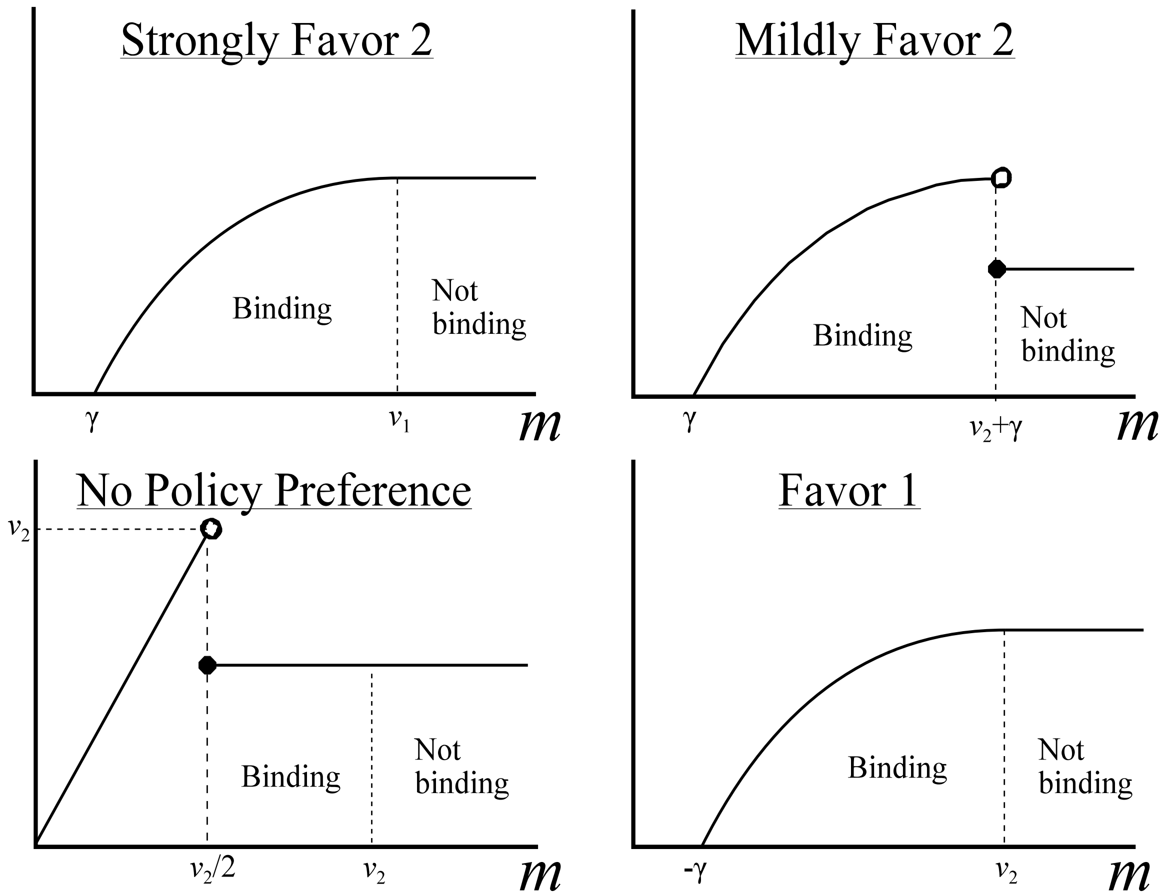

Figure 2: Expected Aggregate Contributions with a Cap

Proposition 2: *Imposition of a cap will lead to an increase in expected aggregate contributions if and only if the politician mildly favors the policy position of the low-valuation lobbyist*, $\gamma \in (0, v_1 - v_2]$.

The proof of Proposition 2 is in Appendix B.

As depicted in Figure 2, when the politician has a mild preference for the policy of the low-valuation lobbyist, expected aggregate contributions jump up with the imposition of a binding cap. A similar jump in the probability that the low-valuation lobbyist wins can also be observed in Figure 3 below. The case of mild-preference for the low-valuation lobbyist's policy position is different from the other cases because the imposition of a cap changes the identity of the player who has the advantage in equilibrium. When the cap is not binding and $\gamma \in (0, v_1 - v_2]$, the high-valuation bidder has the advantage in the competition. He can bid slightly higher than $v_2+|\gamma|$ and win for sure. In equilibrium he is able to use this advantage to secure himself a positive expected payoff, competing away all of the low-valuation bidder's surplus. However, when the contribution cap becomes binding, m falls below $v_2+|\gamma|$, the roles are reversed. Now the high-valuation bidder is effectively constrained. Hence the low-valuation bidder has the option of bidding just above $m-|\gamma|$ guaranteeing victory and a positive payoff. This advantage induces the low-valuation lobbyist to bid more aggressively in equilibrium. This results in a discrete increase in expected aggregate contributions[13] (see Figure 2) and in the probability that the policy of the low-valuation lobbyist gets enacted (see Figure 3). The contribution cap does not change the basic nature of the competition – equilibrium is still in mixed strategies – but it swings the advantage from the high-valuation bidder to his rival whose policy is favored by the politician. Such a reversal does not arise in cases where the politician favors the high-valuation bidder, nor where the low-valuation bidder is strongly favored, and hence expected aggregate contributions are continuous in those cases.[14]

Proposition 3: *For all $\gamma \neq 0$, a more restrictive cap (decreasing m) always reduces the influence of monied interests in policy making; a more restrictive cap increases the probability for winning of the lobbyist whose policy position is preferred by the politician no matter whether it is the high-valuation or the low-valuation lobbyist.*

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[13]The size of the jump is inversely related to the intensity of the preference. With a non-binding cap the low-valuation lobbyist is at a disadvantage due to his low-valuation of the prize. The weaker the preference for his position the greater his disadvantage. Introducing a binding cap tilts the playing field in his favor. Hence when the preference for his position is very mild the introduction of a binding cap makes a big difference. From a playing field titled very much in favor of the high-valuation lobbyist, the low-valuation lobbyist now enjoys a playing field where he has the advantage. So the milder the politician's preference for his policy the greater the change in the aggressiveness of the low-valuation lobbyist, leading to a greater jump in aggregate contributions when the cap becomes binding.

[14]While it seems natural to model the politician's allocation rule as an additive preferential treatment, alternative specifications exist, such as the multiplicative preferential treatment in Fu (2006). However, as long as the lobbyist with the preferred policy can win the prize with a lower contribution than his rival's, a binding cap will effectively constrain only the lobbyist with the less preferred policy. Hence the favored lobbyist will have the advantage due to the cap. Whenever the politician mildly prefers the policy of the low valuation lobbyist, the introduction of a cap will switch the identity of the lobbyist with the advantage. Hence the results in Propositions 1 through 3 are likely to hold for any reasonable specification of politician preferences. Nevertheless, in this context an additive specification has the desirable property that the politician's preference for a policy does not depend on the contributions he receives.

The proof of Proposition 3 and the derivation of expected payoffs and probability of winning are in Appendix B.

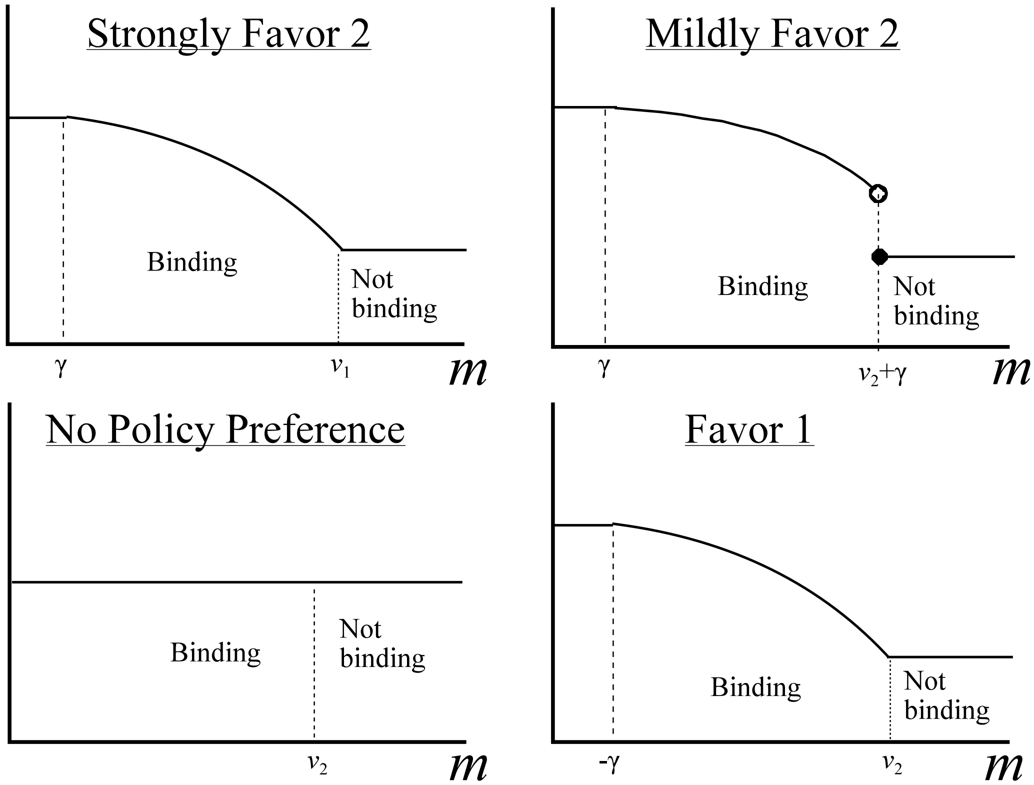

Figure 3: The probability that the politician's favored policy is enacted

Figure 3 graphs the probability that the favored lobbyist wins. Since a more restrictive binding cap tilts the playing field in favor of the lobbyist with the preferred policy alternative, it always makes it more likely that the policy preferred by the politician is enacted. If the politician mildly favors the low-valuation lobbyist's position there is a jump in the probability of the low-valuation policy being enacted at the point where the cap just becomes binding. The intuition of this jump was discussed earlier with Proposition 2.

Contribution caps can be expected to lower special interest group influence, as well as money in politics. A more restrictive cap makes it more likely that the politician enacts the policy alternative he would have enacted if there were no contributions (see Figure 3). Note this measure of the degree of influence of monied interest has an advantage over using expected aggregate contributions in that it captures the concern that policy may be driven by money. The imposition of a binding cap can lead to an increase in the expected aggregate contributions (Figure 2, mild preference for the low-valuation lobbyist's policy) while at the same time leading to an increase the probability that the politician goes with his conscience (Figure 3).

Also note that in equilibrium both lobbyists have a probability mass at zero (see Figure 1). The more restrictive the cap, the more likely it is that the politician does not receive any funds from either lobbyist. In that case the politician simply enacts his preferred policy. A more restrictive cap fosters an environment where it is less likely that special interest group money exerts influence on policy decisions.

Furthermore the politician is likely to have different intensities of policy preference across issues. For all policy issues where the preference is too strong, $|\gamma| \geq m$, lobbyists do not contribute and the politician simply goes with his conscience. A more restrictive cap implies a lower critical threshold of politician preference where there will be no influence of special interest groups on policy making. Hence politician decisions will be swayed by monied interests on a smaller number of questions. A more restrictive binding cap implies decreased expected aggregate contributions on issues where lobbying matters and it implies that there will be less of these policy issues. This suggests that contribution limits can help alleviate Senator Feingold's concern that "only interests with big money to contribute" will be able to effectively petition the legislature.

Propositions 1 and 2 show that a contribution cap always reduces expected aggregate contributions when $|\gamma|$ is sufficiently large. However, when the politician has a mild preference for the policy of the low-valuation lobbyist (small but positive γ), the imposition of a cap can have the unintended consequence of increasing contributions. One interpretation of $|\gamma|$ is the politician's expected future campaign costs required to offset the effect of taking a policy position that is unpopular in his district. Under this interpretation, the effect of a contribution cap on aggregate contributions can be quite different for House members versus Senators, as well as for members from cities versus members from rural areas. Between congressional districts there are vast differences in the cost of communicating with constituents even though they represent the same number of voters. Stratmann (2009) finds that the cost of reaching 1% of constituents with TV advertising during prime time in the 2000 election cycle ranged from \$18 in Idaho's 2nd district to \$1875 in New York City.

Since a politician from a larger or a more urban district is likely to face a higher cost of communicating with constituents, for the same underlying policy preference, the $|\gamma|$ for this politician is likely to be higher. So the cap on contributions may change the distribution of contributions between politicians. It may result in reduced contributions to Senators from larger states but increased contributions to Representatives from districts contained within minor media markets. When states consider contribution caps for state level offices, the experience with national level contribution caps may not directly apply to state politicians who generally have much lower costs of communicating with constituents.

5. Discussion

While there are caps on political lobbying in the U.S., there are no limits on campaign expenditures. Expenditure limits were struck down by the 1976 Supreme Court ruling on Buckley *v.* Valeo as limitations on free speech. There are, however, many countries where there are expenditure limits in place such as the UK, Canada, France and Israel. One of the arguments in support of expenditure limits is that without such limits larger parties would have an unfair advantage over smaller parties. While our model is not tailored for expenditure limits, one may suggest some possible interpretations of the variables that might help shed light on this discussion.

Assume that there are two types of voters: party-loyal voters and swing voters. The swing voters are swayed by campaign spending while the party-loyal voters are not.[15] The party with fewer loyal voters has to spend more in order to win more than 50% of the total votes. If the larger party tends to have more party loyal supporters, then it is subject to “preferential” treatment in the all-pay auction election game. Proposition 3 shows that a cap always increases the probability that the favored bidder wins. Thus the model may suggest that a cap on campaign expenditure (the bids of the political parties to win the election) may in fact benefit the larger party rather than the smaller party, contrary to one of its intended consequences.

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[15]An alternative representation of swing voters is in Kovenock and Roberson (2008) where they are swayed by promises of redistributive policy. Konrad (2004) has a formal model of campaigning and voter behavior.

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APPENDIX A: PROOF OF LEMMA 1 AND LEMMA 2

The case where γ=0 has been extensively studied (see Hillman and Riley ,1989 and Baye *et al.*, 1993 and 1996 without a cap and Che and Gale 1998 with a cap) and so it will be omitted here. Claims 1 through 7 are employed in the proof of Lemma 1 and Lemma 2. Throughout consider just the nontrivial cases where $\gamma \in (-v_2, 0) \cup (0, v_1)$ and $m>|\gamma|$. Define z=min(v_u,m). If there is no contribution cap $z=v_u$.

Claim 1: *Bidder u will not put a probability mass on any level of contribution greater than zero. Without a contribution cap, bidder f will not put a probability mass on any level of contribution greater than zero. With a binding contribution cap, bidder f will not put a probability mass on any bid* $b_f \in (0, m-|\gamma|)$*. With or without a contribution cap, there is no equilibrium in pure strategies.*

Proof: Bidder u will never bid more than z. Suppose the lowest mass point of bidder u in the range B_u=(0, z] is given by $b_u' \in B_u$. Then bidder f would not put any probability at $b_f' = b_u' - |\gamma|$, as a slight increase in his bid would result in a discrete increase in the probability of winning. As there is no probability of b_f' exactly, bidder u could lower his bid slightly without changing his probability of winning. Since bidder u will never bid more than z and since he has no probability mass a z by the above argument, bidder f can win for sure with a bid of z-|γ| so he will never bid more than that. Define a range B_f as B_f=(0, z-|γ|] if $z=v_u$ and as B_f=(0, z-|γ|) if $z<v_u$. Suppose the lowest mass point of bidder f in B_f is given by $b_f'' \in B_f$. Bidder u would not put any probability at $b_u'' = b_f'' + |\gamma|$ since bidding $b_u'' = b_f'' + |\gamma| + \varepsilon$ would yield a discrete increase in probability. So bidder f would prefer a slightly lower bid than b_f''. Both players' bidding zero cannot be sustained as a pure-strategy equilibrium either, since the best response to b_f=0 would be to bid slightly higher than |γ|.

□

Claim 2: *With or without a contribution cap, bidder u will put zero probability on* $b_u \in (0, |\gamma|]$.

Proof: If bidder u contemplates $b_u \in (0, |\gamma|)$ a bid of zero will win with the same probability as he must exceed his rival's bid by at least $|\gamma|$ in order to win. If $b_u = |\gamma|$ then he can win only if b_f=0, in which case there is an even chance of winning. If $b_u = |\gamma|$ gives bidder u nonnegative payoff he could double his chances of winning by a slight increase in his bid. And if $b_u = |\gamma|$ gives him a negative payoff he could get a zero payoff by dropping his bid to zero. □

Claim 3: *If there is a binding contribution cap, or if $\gamma \in (-v_2, 0) \cup (v_1 - v_2, v_1)$ without a contribution cap, bidder u has an infimum bid of zero, and* EV_u=0.

Proof: Bidder *u* would never bid higher than *z*. Bidder *u*'s infimum bid must be less than *z* since there can be no probability mass at *z* by Claim 1. Suppose that bidder *u* has an infimum bid of $b_u' \in (|\gamma|, z)$. Then bidder *f* would never choose $0 < b_f \leq b_u' - |\gamma|$. If he did he would be paying a positive amount and would lose for sure, since the probability of bidder *u* choosing exactly b_u' is zero by Claim 1. Therefore bidder *u* could lower his bid without changing the probability of winning. Suppose that bidder *u*'s infimum bid is $b_u' = |\gamma|$ where bidder *u* is mixing in the open interval above |γ| but not at |γ|, by Claim 2. Then bidder *f* would never bid zero as this would give a zero payoff and he can win for sure with a bid of *z*-|γ|+ε yielding a positive payoff. Take a bid of $b_u = |\gamma| + \varepsilon$, the probability that bidder *u* wins with this bid is $\int_{|\gamma|^+}^{|\gamma|+\varepsilon} f_f(x - |\gamma|)dx$. Since bidder *f* has no mass point on (0, ε] by Claim 1, this probability is close to zero for small ε, yielding a negative expected payoff for bidder *u*. Hence bidder *u*'s infimum bid cannot be |γ|. $b_i^{\inf} \in (0, |\gamma|)$ is not possible by Claim 2. Therefore $b_u^{\inf} = 0$. At this bid he loses for sure, so EV_u =0. □

Claim 4: *If there is a binding contribution cap, or if $\gamma \in (-v_2, 0) \cup (v_1 - v_2, v_1)$ without a contribution cap, bidder u has a suprimum bid of z. Bidder f has a suprimum bid of z-|γ| and* $EV_f = v_f + |\gamma| - z > 0$.

Proof: Suppose that bidder *u* has a suprimum bid of $b_u' < z$. Then bidder *f* would never set $b_f > \max[0, b_u' - |\gamma|]$ as he can win for sure with $b_f = \max[0, b_u' - |\gamma|]$ since the probability of bidder *u* choosing exactly b_u' is zero by Claim 1. Therefore bidder *u* could win for sure with $b_u = b_u' + \varepsilon$ yielding a payoff greater than zero for small enough ε, a contradiction of Claim 3. Hence the suprimum bid of *u*, $b_u^{\sup} = z$. Suppose that bidder *f* had a suprimum bid of $b_f' < z - |\gamma|$. Then bidder *u* could win for sure with $b_u = b_f' + |\gamma| + \varepsilon$ yielding a payoff greater than zero for small enough ε, a contradiction of Claim A3. B*f* can win for sure with a bid in the open interval above $z - |\gamma|$ hence $b_f^{\sup} = z - |\gamma|$. Since $z - |\gamma|$ is in the support of *f*'s mixed strategy and he wins for sure with that bid, $EV_f = v_f + |\gamma| - z$. □

Claim 5: *Without a contribution cap, if $\gamma \in (0, v_1 - v_2]$, u=1 and f=2, bidder u has an infimum bid of γ. Bidder f has an infimum bid of zero and EV_f=0.*

Proof: Bidder *u* can win for sure with a bid of $v_f + \gamma$ yielding a payoff of $v_u - v_f - \gamma > 0$. He would never bid zero since he would lose for sure. $b_u^{\inf} \in (0, \gamma)$ is not possible by Claim 2. $b_u^{\inf} = v_f + \gamma$ would be a pure strategy, but Claim 1 establishes that there is no pure-strategy Nash equilibrium. Suppose that bidder *u* has an infimum bid of $b_u' \in (\gamma, v_f + \gamma)$. Then bidder *f* would never choose $0 < b_f \leq b_u' - \gamma$. If he did he would be paying a positive amount and would lose for sure, since by Claim 1, the probability of bidder *u* choosing exactly b_u' is zero. Therefore, bidder *u* could lower his bid without changing the probability of winning. Hence $b_u^{\inf} = \gamma$. Suppose bidder *f* had an infimum bid of $b_f' \in (0, v_f]$, then bidder *u* would never choose $b_u \leq b_f' + \gamma$. If he did, bidder *u* would lose for sure yielding a negative payoff. Since by Claim A1 the probability of bidder *f* choosing exactly b_f' is zero and bidder *u* can always guarantee a positive payoff of $v_u - v_f - \gamma > 0$. But then bidder *f* would prefer a bid of zero to b_f'. Therefore $b_f^{\inf} = 0$. At this bid he loses for sure, so EV_f=0. □

Claim 6: *Without a contribution cap, if $\gamma \in (0, v_1 - v_2]$, u=1 and f=2 and bidder u has a suprimum bid of $v_f + |\gamma|$ and $EV_u = v_u - v_f - \gamma > 0$. Bidder f has a suprimum bid of v_f.*

Proof: Given that *f* would never bid higher than his valuation of the prize, *u* would never bid higher than $v_f + |\gamma|$. Suppose that bidder *u* had a suprimum bid of $b_u' < v_f + |\gamma|$. Then bidder *f* would never set $b_f > \max[0, b_u' - |\gamma|]$ as he can win for sure with $b_f = \max[0, b_u' - |\gamma|]$ since the probability of bidder *u* choosing exactly b_u' is zero by Claim 1. Therefore bidder *f* could win for sure with $b_f = b_u' - |\gamma| + \varepsilon$ yielding a payoff greater than zero for small enough ε, a contradiction of Claim 5. So $b_u^{sup} = v_f + |\gamma|$. By Claim 1, bidder *u* wins for sure with a bid of $v_2 + |\gamma|$, so $EV_u = v_u - v_f - |\gamma| > 0$. Suppose that bidder *f* has a suprimum bid of $b_f' \in (0, v_f)$. Then bidder *u* would never set $b_u > b_f' + |\gamma|$ since he could win for sure with $b_u = b_f' + |\gamma|$ given that probability that bidder *f* chooses b_f' exactly is equal to zero by Claim A1. Therefore bidder *f* could win for sure with $b_f = b_f' + \varepsilon$ yielding a payoff greater than zero for small enough ε, a contradiction of Claim 5. So $b_f^{sup} = v_f$. □

Claim 7: *For bidder u bids almost everywhere on $b_u \in (b_u', b_u'']$ and for bidder f, bids almost everywhere on $b_f \in (b_f', b_f'']$ must have positive probability, where*

if there is no contribution cap:

$\forall \gamma \in (v_1 - v_2, v_1) \cup (-v_2, 0)$	$b_u' = \lvert\gamma\rvert,\ b_u'' = v_u$	*and* $b_f' = 0,\ b_f'' = v_u - \lvert\gamma\rvert$
$\forall \gamma \in (0, v_1 - v_2]$	$b_u' = \lvert\gamma\rvert,\ b_u'' = v_f + \lvert\gamma\rvert$	*and* $b_f' = 0,\ b_f'' = v_f$

if there is a contribution cap:

	$b_u' = \lvert\gamma\rvert,\ b_u'' = m$	*and* $b_f' = 0,\ b_f'' = m - \lvert\gamma\rvert$

Proof: Suppose there were an interval (t, s) in (b_u', b_u'') where bidder *u* had zero probability of bidding. Then bidder *f* would have zero probability of bidding in (t-$|\gamma|$, s-$|\gamma|$) since he could lower his bid to t-$|\gamma|$ and have the same chance of winning. But in this case bidder *u* would never bid $s + \varepsilon$ as he could lower his bid to t, saving $s + \varepsilon - t$ in bidding costs and losing only $F_f(s + \varepsilon - |\gamma|) - F_f(t - |\gamma|)$ in probability. By Claim 1 the loss in probability is negligible for small ε. So if there were an interval of zero probability it must go up to b_u'', which depending parameter values contradicts either Claim 4 or Claim 6. A symmetric argument rules out ranges of zero probability for bidder *f* on $b_f \in (b_f', b_f'']$. □

PROOF OF LEMMA 1: ***Characterization of the equilibrium without cap***

(i) $\gamma \in (-v_2, 0) \cup \gamma \in (v_1 - v_2, v_1)$. Claims 1, 2, 3,4 and 7 show that bidder *u* must be indifferent among all bids almost everywhere in $\{0\} \cup (|\gamma|, v_u]$ and bidder *f* is indifferent among all bids almost everywhere in $[0, v_u - |\gamma|]$. EV_u =0 by Claim 3. Bidder *u* wins the prize v_u when he bids $b \in (|\gamma|, v_u]$ only with the probability that bidder f contributes less than b-|γ|. Hence, v_u $F_f(b-|\gamma|)-b = 0$. So, $F_f(b)=(b+|\gamma|)/v_u$ $\forall$ b $\in[0,v_u-|\gamma|]$. Bidder f has a probability mass equal to $|\gamma|/v_u$ at zero. $EV_f = v_f+|\gamma|-v_u$ by Claim 4. Bidder f wins the prize v_j when he bids $b \in[0,v_u-|\gamma|]$ only with the probability that bidder u does not exceed bidder f's bid by more than |γ|: So the indifference implies $v_f F_u(b+|\gamma|)-b = v_f-v_u+|\gamma|$. Hence $F_u(b)= (v_f-v_u+b)/v_f$ $\forall$ b $\in(|\gamma|,v_u]$. Bidder u has a probability mass equal to $(v_f-v_u+|\gamma|)/v_f$ at zero. And he puts zero probability on $(0,|\gamma|]$ by Claim 2.

(ii) $\gamma\in(0,v_1-v_2]$. In this case f=2 and u=1. Claims 1, 2, 5, 6, and 7 show that bidder u is indifferent between bids almost everywhere in $(\gamma,v_f+\gamma]$ and bidder f is indifferent between bids almost everywhere in $[0,v_f]$. $EV_u= v_u-v_f-\gamma$, by Claim 6. Bidder u wins the prize v_u when he bids $b \in (\gamma,v_f+\gamma]$ only if bidder f bids less than b-γ. Therefore v_u $F_f(b-\gamma)-b=v_u-v_f-\gamma$. So, $F_f(b)=(v_u-v_f+b)/v_1$ $\forall$ $b \in[0,v_f]$. Bidder f has a probability mass of $(v_u-v_f)/v_u$ at zero. EV_f=0 by Claim 5. Bidder f wins the prize v_f when he bids $b \in [0,v_f]$, only if bidder u bids less than b+γ. So, v_f $F_u(b+\gamma)-b=0$. Therefore $F_u(b)=(b-\gamma)/v_f$ $\forall$ $b \in(\gamma,v_f+\gamma]$. Bidder u puts zero probability on $(0,\gamma]$ by Claim 2. □

PROOF OF LEMMA 2: ***Characterization of the equilibrium with a cap***

Claims 1, 2, 3,4 and 7 demonstrate that in equilibrium bidder u is indifferent among all bids almost everywhere in $\{0\}\cup(|\gamma|, m]$ and bidder f is indifferent among all bids almost everywhere in $[0,m-|\gamma|]$. EV_u =0 by Claim 3. Bidder u wins the prize v_u when he bids $b \in(|\gamma|,m]$ only if the bidder who's policy if favored bids less than b-|γ|. Hence, v_u $F_f(b-|\gamma|)-b = 0$. So, $F_f(b)=(b+|\gamma|)/v_u$ $\forall b\in[0,m-|\gamma|]$. Bidder f has a probability mass equal to $|\gamma|/v_u$ at zero. The equilibrium distribution function is discontinuous. There is a probability mass equal to $1-F_f(m-|\gamma|)=1-m/v_u$ in the open interval above m-|γ|. $EV_f = v_f + |\gamma| - m$ by Claim 4. Bidder f wins the prize v_f when he bids $b\in[0,m-|\gamma|]$ only with the probability that bidder u does not exceed bidder f's bid by more than |γ|: $v_f F_u(b+|\gamma|)-b = v_f+|\gamma|-m$. So, $F_u(b)= (v_f-m+b)/v_f$ $\forall$ $b \in(|\gamma|,m]$. Bidder u has a probability mass equal to $(v_f-m+|\gamma|)/v_j$ at zero. There is a gap in the support of equilibrium bids. By Claim 2 bidder u puts zero probability on $(0,|\gamma|]$.

□

APPENDIX B: PROOF OF PROPOSITIONS

PROOF OF PROPOSITION 1: ***Change in expected aggregate contributions w.r.t. binding cap***

On $b \in(|\gamma|,m]$ the p.d.f. of the bids of bidder u is $f_u(b)=1/v_f$. The expected contribution of bidder u is

$$\int_{b_i-|\gamma|^+}^{m} x f_i(x)dx = \frac{m^2-\gamma^2}{2v_f}$$

On $b \in (0, m-|\gamma|]$ the p.d.f. of bidder f's bids is $f_f(b)=1/v_u$. The expected contribution of bidder f is

$$\int_{b_f=0}^{(m-|\gamma|)} x f_f(x)dx + (m-|\gamma|)(1-m/v_u) = \frac{(m-|\gamma|)}{2v_u}(2v_u - m - |\gamma|)$$

The derivative of expected aggregate contributions with respect to m is equal to $\left[(m/v_f)+(v_u-m)/v_u\right]$. This term is positive since when $\gamma \in (-v_2, 0) \cup \gamma \in (v_1-v_2, v_1)$ a binding cap is $m<v_u$ and when $\gamma \in (0, v_1-v_2]$ $u=1$ and $f=2$ so $v_u>v_f$. □

PROOF OF PROPOSITION 2: ***Change in expected aggregate contributions due to imposition of a binding cap***

See Section 3 in the main text for the derivation of expected contributions when there is no cap. See the proof of Proposition 2 above for expected aggregate contributions when there is a binding cap. Evaluate expected aggregate contributions with a binding cap where the cap just becomes binding. When $\gamma \in (-v_2,0) \cup (v_1-v_2,v_1)$ expected aggregate contributions is continuous at the point where the cap becomes just binding ($m=v_2-\varepsilon$) and it equal to $\left[(v_2^2-|\gamma|^2)/2v_1+(v_2-|\gamma|)^2/2v_2\right]$ as $\varepsilon \to 0$. However when $\gamma \in (0,v_1-v_2]$, expected aggregate contributions is discontinuous. The expected aggregate contributions with no cap are equal to $\left[(v_2+2\gamma)/2+v_2^2/2v_1\right]$. The expected aggregate contributions with a binding cap where the cap just becomes binding ($m=v_2+|\gamma|-\varepsilon$) is equal to $\left[(v_2+2|\gamma|)/2+v_2(2v_1-v_2-2|\gamma|)/2v_1\right]$ as $\varepsilon \to 0$. Hence the imposition of a barely binding cap leads to a discrete jump up in expected aggregate contributions. The size of the jump is equal to $\left[(v_1-(v_2+|\gamma|)v_2/v_1\right]>0$. □

PROOF OF PROPOSITION 3: ***Change in $prob_f$ w.r.t cap***

Bidder u wins the prize with a bid b only with the probability that bidder f does not exceed b-|γ|. By Lemma 1 and Lemma 2 if there is a binding contribution cap, or if $\gamma \in (-v_2,0) \cup (v_1-v_2,v_1)$ without a contribution cap then

$$\text{Prob}_u = \int_{b=|\gamma|}^{z} F_f(x-|\gamma|)f_u(x)dx = \int_{b=|\gamma|}^{z} \frac{x}{v_u v_f}dx = (z^2-\gamma^2)/2v_u v_f$$

Since $z=\min(v_u, m)$, if $\gamma \in (-v_2,0) \cup (v_1-v_2,v_1)$ the probability that the unfavored lobbyist wins is continuous and increasing in m, hence the probability that the favored lobbyist wins is continuous and decreasing in m. Likewise if $\gamma \in (0,v_1-v_2]$ whenever the cap is binding the probability that the favored lobbyist wins is continuous and decreasing in m. In Section 3 it is shown that when $\gamma \in (0,v_1-v_2]$ and there is no cap the probability that bidder u wins the contest is equal to $(1-v_f/2v_u)$. When a barely binding cap is introduced $(m=v_2+|\gamma|-\varepsilon)$ the probability that bidder u wins the contest jumps down to $[(v_f+2|\gamma|)/2v_u]$ where $u=1$ and $f=2$. Hence the discrete increase in the probability that the favored policy position enacted is given by $[(v_u-(v_f+|\gamma|))/v_u]>0$. □