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Ivan Pastine and Tuvana Pastine

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Ivan Pastine, Bilkent University and CEPR
Tuvana Pastine, Bilkent University and CEPR

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Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

Cost of Delay, Deadlines and Endogenous Price Leadership*

This Paper analyses endogenous price leadership in a duopolistic market with differentiated products and symmetrically informed firms. We study the effects of deadlines and discounting in a standard endogenous leadership model. We show that there will be occasional changes in the identity of the price leader with any cost of delay or discounting, however small. By analysing the incentives that induce a firm to take up the leader position we derive positive predictions about which firm will lead most price changes.

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Tuvana Pastine
Economics Department
Bilkent University
06533 Bilkent
Ankara
TURKEY
Tel: (90 312) 290 1598/1955
Fax: (90 312) 266 5140
Email: tuvana@bilkent.edu.tr

Ivan Pastine
Economics Department
Bilkent University
06533 Bilkent
Ankara
TURKEY
Tel: (90 312) 290 1598/1955
Fax: (90 312) 266 5140
Email: pastine@bilkent.edu.tr

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I. Introduction

Case studies find that in a wide variety of oligopolistic industries, such as the cigarette, steel, automobile, ready-to-eat-cereal, and gasoline industries, new price announcements arrive in a sequential manner; price increases by one firm are followed immediately by its rivals. Compared with simultaneous price competition this leader-follower time pattern of pricing typically yields higher prices for both the leader and the follower firm. Hence, it is difficult for regulatory authorities to distinguish collusive price leadership from a leader-follower pattern that emerges as a non-cooperative equilibrium outcome.

This paper inquires into the types of outcomes that are likely to arise as the result of non-cooperative competition between similar, symmetrically informed firms.¹ First, we show that in such industries there will be occasional changes in the identity of the price leader. A leader-follower pattern where a single firm consistently leads all price changes is unlikely if firms face a cost of delay in their price announcements. Secondly, by analyzing the incentives that induce a firm to take up the leader position we derive positive predictions about which firm will lead most price changes.

Indeed, in many industries, empirical studies by Scherer and Ross (1990), Nicholls (1951) and Markham (1951) show that the identity of the leader tends to vary. One of the “distinguishing characteristics” of price leadership in industries that do not have a dominant firm is occasional changes in the identity of the leader firm (Scherer and Ross, 1990, pp. 249). For instance, in the U.S. cigarette industry from 1923 to 1941 there were eight standard brand price changes. While Reynolds led six of these price changes, American led the other two price changes. In the 1960's steel industry, price leadership passed from one company to another with price changes being announced in different product lines. Markham (1951) reports that in the newsprint industry, while International Paper led most price changes in markets east of the Rocky Mountains, it did not lead all of the price changes. Likewise, Crown Zellerbach, has usually announced new prices on the west coast, but its rivals lead some price changes as well.

¹Rotemberg and Saloner (1990) and Eckard (1982) analyze the role of informational asymmetries in endogenous price leadership.

There is a line of research where theory predicts a single firm as the price leader of the industry. Among these, Van Damme and Hurkens (1998) show that an equilibrium refinement (risk dominance) will pick the low-cost firm as the price leader no matter how similar the firms might be, as long as they are not identical. In Deneckere and Kovenock (1992) and Deneckere, and Kovenock and Lee (1992) differences in capacity constraints and brand loyalty can generate an endogenous price leader as a result of a pure-strategy equilibrium. The findings of our paper, on the other hand, predict occasional changes in the identity of the leader. The reasons for the difference in these predictions are two-fold: We explicitly incorporate a cost of delay in price announcements and we allow the firms to act whenever they wish.

When firms are restricted in the timing of their moves such that in odd periods one firm can announce price and in even periods the other firm can announce price, Deneckere and Kovenock (1992) and Deneckere, and Kovenock and Lee (1992) show that the high capacity firm and/or the firm with a greater brand loyal group of consumers will emerge as the endogenous price leader. However as also noted in Deneckere and Kovenock (1992), relaxing the restriction on the timing of the moves such that each firm can move in any period, endogenous price leadership as a pure-strategy equilibrium outcome disappears when firms discount the future. Our framework extends the endogenous leadership discussion by examining the nature of the competition under these circumstances.

We argue that a cost of delay in price announcement may play an important role in firms' strategic pricing decisions. A firm can collect sales receipts at the new prices only after having announced its price. Hence a delay in the price announcement will inevitably result in delayed, and therefore discounted profits for the firm. When firms incur any cost of delay in price announcements, however small, a leader-follower pattern cannot emerge as a result of a pure-strategy Nash equilibrium.

Hence, we examine the mixed-strategy equilibria. In price competition the mixed-strategy equilibrium is a war of attrition. Due to the basic conflict over role selection firms struggle to capture the advantageous follower position. In the war of attrition equilibrium,

asymmetries between the firms translate into asymmetric probabilities of leading. Therefore while one firm might lead many price changes, the same firm will not lead all price changes.

We also explore the effect of deadlines for price announcements. The pure-strategy leader-follower equilibria disappear with any cost of delay or discounting, however small and mixed-strategy equilibrium is shown to break down with a deadline. The only non-cooperative equilibrium that survives is simultaneous price competition. Therefore a leader-follower pattern is unlikely to occur as a result of non-cooperative competition when firms face a strict deadline for price announcements.

II. A Standard Model of Endogenous Price Leadership

Consider a duopolistic market with differentiated products where risk-neutral profit-maximizing firms, A and B compete in prices.² There are two periods and each firm must make a price commitment either in period one or in period two. In period one, firms simultaneously decide whether to commit and if so what price to commit to, not observing each others decision. If the firm waits and its rival commits, the firm can observe rival's commitment price while setting its price in the second period. If both firms wait, they set their prices simultaneously in period two. Consumers make their purchasing decisions as soon as both firms announce their prices. This framework corresponds to the extended game with action commitment model of Hamilton and Slutsky (1990) which has been widely used to study endogenous price leadership.³ In this framework each firm must announce its price at the latest in the second period (at the deadline). It will turn out that the equilibrium in the timing of moves and the leader-follower predictions depend critically on the existence of a deadline.

In this paper we will follow much of the endogenous leadership literature in assuming that firms are able to credibly commit to their prices: Once a firm announces its price it cannot change it. While this is a common assumption in the literature, it does imply that the framework is far from a complete characterization of the firms' strategic problem. Nevertheless, in the

²Holthausen (1979) stresses the importance of the degree of risk aversion in endogenous price leadership models.

³See also Amir (1995). See Hamilton and Slutsky (1990) for a formal definition of the game.

absence of a device that allows firms to credibly commit to prices, they would have an incentive to try to find such a device. Without the ability to commit firms would engage in simultaneous price competition which typically yields lower profits than the firms would obtain in equilibrium with a commitment device.⁴ Although *ex ante* firms would like to find a commitment device, it is also true that *ex post* they would like to break their commitment. For a complete explanation of observed price leadership more work needs to be done to understand how firms may address the commitment problem. Not only is this an important issue in its own right, but the way in which firms attempt to commit may also affect the nature of the competition between them, and hence their struggle over leader and follower roles. However, here we will abstract from these important issues and confine ourselves to the simpler problem of firm incentives taking the existence of a commitment device as given.

Looking at payoffs, each firm has a twice differentiable, strictly concave profit function which is increasing in the rival's price. Given firm j 's price p_j , firm i 's optimal price solves the first order condition $\frac{\partial \Pi_i(p_i, p_j)}{\partial p_i} = 0$, where $i, j \in \{A, B\}$ $i \neq j$. These equations implicitly define reaction functions, $p_i = R_i(p_j)$ which are assumed to be upward-sloping.⁵ It is also assumed that there exists a unique and stable set of "Simultaneous" prices where the reaction functions cross, $p_i^S = R_i(p_j^S)$.⁶ The profit functions evaluated at these simultaneous prices are denoted by Π_i^S , $i \in \{A, B\}$. The profit of firm i when it is the leader is given by $\Pi_i(p_i, R_j(p_i))$, which is assumed to be strictly concave. The optimal "Leadership" price is denoted by p_i^L . The reaction function of the follower firm j , evaluated at p_i^L yields firm j 's optimal "Follower" price, $p_j^F = R_j(p_i^L)$. The

⁴Work by Henckel (1999) shows one method of endogenizing the commitment level.

⁵Upward-sloping reaction functions, which are typically assumed in price competition, require that the cross partial derivatives of the profit functions are positive.

⁶Stability implies that at the simultaneous prices $\frac{\partial R_B}{\partial p_A} < 1 / \frac{\partial R_A}{\partial p_B}$. This standard assumption on the slopes of the reaction functions is used in Lemma 1 to ensure that in equilibrium firms will never charge less than their simultaneous price or more than their Stackelberg leader price.

optimized leadership profit $\Pi_i(p_i^L, p_j^F)$ is denoted by Π_i^L .⁷ The optimized follower profit $\Pi_j(p_i^L, p_j^F)$ is denoted by Π_j^F .

For the purposes of this paper we will define a leader-follower outcome to be any outcome that may possibly help to explain the empirically observed phenomenon of price leadership. That is any outcome where one firm announces its price first and the other announces its price an observable (non-zero) amount of time after the leader. A leader-follower equilibrium will be any equilibrium which yields a non-zero probability of a leader-follower outcome. In the context of this two-period discrete-time model, a leader-follower outcome would involve one firm moving in period one, and the other moving in period two.

Hamilton and Slutsky (1990, theorem VII) show that in this framework, multiple subgame-perfect pure-strategy leader-follower Nash equilibria exist including each firm waiting and the other playing its Stackelberg leader price in the first period. The third and last pure-strategy subgame-perfect Nash equilibrium is where both firms announce their simultaneous prices in the first period.

In Hamilton and Slutsky (1990) firms do not discount the future, and they do not face a cost of delay in their price announcements. However, costs of delay arise naturally in price announcement games. Firms will not receive the benefit of new prices until they announce their price, in which case discounting will introduce a cost of delay. Additionally, firms may incur costs due to foregone sales in the face of late price quotations. When consumers are unable to preplan due to lack of information about price, demand may decrease.

In fact, pure-strategy leader-follower equilibria disappear if firms incur any cost of delay in price announcements, however small the cost might be.⁸ When i expects j to commit to p_j in period one, it is in i 's best interest to quote $R_i(p_j)$. With any cost of delay, however small, i

⁷ When the leadership first order condition, $\frac{\partial \Pi_i}{\partial p_i} + \frac{\partial \Pi_i}{\partial p_j} \frac{\partial R_j}{\partial p_i} = 0$, is evaluated at the simultaneous price the left-hand side is positive since the profit function is increasing in the rival's price and the reaction function is positively sloped. Therefore, $p_i^L > p_i^S$. While the leader firm has the option of quoting its simultaneous price and receiving its simultaneous profit, it will not choose to do so, hence $\Pi_i^L > \Pi_i^S$.

⁸See Deneckere and Kovenock (1992) footnote 12. In Robson (1990) sales take place at the deadline, no matter how early the firms might announce their prices. Since there is no cost of delay in price announcements in his framework, Robson finds that there are multiple pure-strategy leader-follower equilibria.

strictly prefers to quote $R_i(p_j)$ in period one rather than in period two, since given j 's first period price i can capture the same profit in the first period and avoid the cost of delay. Hence j leading and i following is not an equilibrium when firms face a cost of delay, however small the cost of delay. This result suggest that the conclusions of the pure-strategy leader-follower equilibria are unlikely to be a convincing explanation for most observed price leadership.

Nevertheless, empirical observations suggest that price leadership is quite common. One reasonable interpretation could have been that firms are playing a mixed-strategy equilibrium, randomizing between committing early and waiting to announce price. The literature has noted the possibility of such an equilibrium (see Hamilton and Slutsky,1990, Robson,1990, and Deneckere and Kovenock,1992).⁹ In this context it would be intuitively appealing. Dowrick (1986) shows that both firms will typically prefer to be the *follower* to being the leader in price competition when firms have upward-sloping reaction functions.¹⁰ Dowrick notes that it may also be possible for firms with sufficiently dissimilar profit functions to agree on the choice of roles. We will concentrate on the cases where firms are not too dissimilar, so there is an element of conflict in role selection. We restrict attention to the set of profit function pairs which yield leadership prices above the follower prices, $p_i^L > p_i^F$. Since, by definition, a firm's follower price is its best response to its rival's leader price, it follows that $\Pi_i(p_i^F, p_j^L) \geq \Pi_i(p_i^L, p_j^L)$. And since the profit function is increasing in the rival's price, $\Pi_i(p_i^L, p_j^L) > \Pi_i(p_i^L, p_j^F)$. Therefore for the set of profit function pairs where the leader prices are above the follower prices there will be an element of conflict over the role selection and both firms will want to capture the follower position. In this case, in a model with a deadline a mixed-strategy equilibrium would be interpreted as the "Chicken" variation of the war of attrition.¹¹

⁹See also Pal (1996) for a model where similar equilibria arise in a quantity competition setting. Tirole (1988, pg. 332) notes that such a mixed-strategy equilibrium may arise in the closely related issue of the application of price-protection policies.

¹⁰Also see Gal-Or (1985). Amir and Grilo (1997), Boyer and Moreaux (1987) and Ono (1982) discuss closely related questions.

¹¹In the classic game of chicken two cars drive toward a cliff and the first driver to turn away loses (captures the leader profit) while his rival wins (captures the follower profit). However, if neither driver turns away they drive over the cliff (engage in simultaneous price competition).

However, below we show that with a deadline for price announcements a non-degenerate mixed-strategy equilibrium does not exist. Thus with a cost of delay and a deadline there is no equilibrium which yields any chance of a leader-follower outcome.

Claim 1: *With a cost of delay and a deadline for price announcements, the unique subgame-perfect equilibrium involves immediate and simultaneous moves in the first period.*

Proof: As shown above, when there is a cost of delay there is no subgame-perfect pure-strategy leader-follower equilibrium. Lemma 1 in Appendix A shows that firms will never mix in prices. It remains to be shown then that there does not exist an equilibrium where firms randomize over the timing of their price announcements. Suppose there existed a non-degenerate mixed-strategy equilibrium where i commits before the deadline with probability $q_i \in (0,1)$. The commitment price of i is denoted by p_i^C . In Lemma 1 we show that $p_i^C \in (p_i^S, p_i^L)$.¹² If j waits until the deadline, with probability q_i it captures the follower position. With probability $(1 - q_i)$, however, firms quote prices simultaneously right at the deadline. Therefore, j 's expected profit from waiting is,

$$q_i \Pi_j(p_i^C, R_j(p_i^C)) + (1 - q_i) \Pi_j^S \quad (1)$$

Firm j 's expected profit from quoting $R_j(p_i^C)$ before the deadline is,

$$q_i \Pi_j(p_i^C, R_j(p_i^C)) + (1 - q_i) \Pi_j(R_i(R_j(p_i^C)), R_j(p_i^C)) \quad (2)$$

(2) > (1) by Lemmas 1 and 2, in Appendix A. Firm j would always strictly prefer committing to waiting, since j can avoid any chance of the low simultaneous profit at the deadline by committing before the deadline to the price $R_j(p_i^C)$. \square

While endogenous price leadership with a deadline is quite similar to the chicken variation of the war of attrition, it differs in one critical aspect. In a classic war of attrition payoffs depend only on the timing of the players' moves. In the price leadership context however, payoffs depend on the prices chosen by the firms. The timing of moves affects the prices, but in the final analysis it is the prices which determine the payoffs. This creates the

¹²This second part of Lemma 1 was suggested, but not proven, by Hamilton and Slutsky (1990).

possibility for deviations which are not possible in the usual applications. In particular, it is possible to leave the game at the same time as your rival and still win. If one firm announces its leadership price and the other announces its follower price at the same time, the firm with the follower price “wins” even though the announcements were simultaneous. This difference leads to a breakdown in the war of attrition equilibrium just before the deadline, and hence a mixed-strategy equilibrium does not exist.

III. Relaxing the Deadline Assumption

The framework presented in the previous section of the paper is welcoming for comparison of the results with the existing literature on endogenous leadership. The introduction of a cost of delay in price announcements, however small, is shown to change the predictions of the model: When there is a strict deadline for price announcements, a leader-follower outcome cannot occur as the result of a non-cooperative equilibrium. However, under this specification the framework no longer has explanatory power for observed price leadership. Since a cost of delay in price announcement games is quite a realistic assumption, how can we reconcile theory and empirical observation? In this section we relax the assumption of strict deadlines for price announcements. While a strict deadline may be a reasonable assumption for some markets, in general it seems somewhat contrived. We show that when firms are not faced with a strict deadline, the theory does generate endogenous leadership and we can make positive predictions about the likely identity of the leader firm.

In the model with no deadline for price announcements we formulate the problem in continuous time. The notation and the assumptions on profit functions in the framework of Section II will carry over to this section. In our price announcement game the continuous-time formulation has two advantages. First, the continuous-time framework allows the firms to act whenever they wish, avoiding arbitrary restrictions on the timing of their actions. Secondly, the equilibrium that yields interesting predictions in the framework without a deadline will prove

to be in mixed strategies and hence the continuous-time formulation is technically convenient.¹³ However, in continuous time when firms can respond instantaneously, leader-follower outcomes are observationally indistinguishable from simultaneous price announcements. In reality firms are observed to respond with a short but perceptible lag, which is what yields the empirical evidence of price leadership. To match this reality we will assume that it takes a fixed time to react to the rival's actions. In the previous section, the assumption of a minimum response time was already imbedded in the discrete-time setting.¹⁴ Hence, in this section we will keep the reaction lag spirit of the discrete-time framework, but allow the firms to act whenever they wish by adopting a continuous-time framework.

It takes a fixed time $m > 0$ to react to the actions of a firm's rival. That is, if a firm wishes to adopt a strategy that conditions its price announcement on its rival's price, then the firm's announcement must occur at least m after that announcement. The firm still has the option of announcing its price earlier, but if it does so then its price cannot be conditional on its rival's price. Sales take place only after both firms have announced their prices. We incorporate a cost of delay in price announcements via discounting. Firms discount the future at the rate $r_i \in (0, \infty)$, $i \in \{A, B\}$. Since p_i^F is the optimal reaction to p_j^L it is always true that $\Pi_i(p_i^L, p_j^L) < \Pi_i(p_i^F, p_j^L)$. We will assume that firms' reaction time is short enough that it does not change this basic incentive of the firms to capture the follower position: $\Pi_i(p_i^L, p_j^L) < e^{-mr_i} \Pi_i(p_i^F, p_j^L)$.

In summary, each of the two firms must make an irrevokable price commitment, which can be done at any time $t \in \mathbb{R}_+$. The actions of the rival are immediately observable, however, if a firm wishes to condition its price on the price chosen by its rival it must announce its price at least time $m > 0$ after the rival's price announcement. The game ends as soon as both firms make

¹³Alternatively one could use a discrete-time framework without a deadline. In that case in the mixed-strategy equilibrium firms must take into account that there is a possibility that they will both commit to a price in the same period (whereas in equilibrium in continuous-time this is a zero-probability event). Thus, in discrete-time the commitment prices and probabilities would be jointly determined, and in general can only be solved implicitly. Therefore the derivation is more tedious and the results are more difficult to compare directly to the existing literature. Nevertheless, the main result that the only equilibrium that can yield a leader-follower outcome is in mixed strategies also holds in the discrete-time framework.

¹⁴See Simon and Stinchcombe (1989) for a comparison of games in discrete-time versus continuous-time and for a clear discussion on the "length of reaction" in these two different frameworks.

their price announcements and each firm receives profits which depend on prices as specified in section II. *Ex ante* each firm i discounts future profits at the rate $r_i > 0$. Attention is restricted to subgame-perfect equilibria.

When there is a deadline for price announcements the result on the non-existence of a mixed-strategy equilibrium remains in continuous time (see Appendix B). Now relax the deadline assumption – firms have the option of delaying their price announcements as long as they like. Simultaneous announcement of p_A^S and p_B^S is an equilibrium outcome in this model, for the same reason as it was in the model with a deadline. Of interest here is what types of leader-follower outcomes are possible.

Claim 2: *Subgame-perfect leader-follower equilibria can only arise due to a mixed-strategy equilibrium where firms randomize over their price-announcement time. There exists such an equilibrium where the probability of a firm announcing its price at time τ , conditional on neither firm having announced its price by time τ , is constant and it is given by $q_j = \frac{r_i \Pi_i^L}{\Pi_i^F - \Pi_i^L}$ $i, j \in \{A, B\}$ and the p.d.f. of the price announcement time follows an exponential distribution $(q_A + q_B) e^{-(q_A + q_B)t}$.*

Proof: Lemma 3 in Appendix A shows that pure-strategy leader-follower equilibria do not exist. Consider potential mixed-strategy equilibria. Firms will never mix in prices by Lemma 1. Consider a possible stationary equilibrium with behavior strategies for each firm j of the form: “If neither firm has announced price by time τ , then announce p_j^L at τ with probability q_j . If the other firm announces price at time τ and j did not, then announce $p_j = R_j(p_i)$ at time $\tau + m$.” If these strategies form a mixed-strategy equilibrium, the probabilities q_A and q_B are such that each firm is indifferent between committing to its leadership price at time τ and waiting a short period of time dt in hopes that the rival commits. For small dt the probability that firm j announces its price in the interval $[\tau, \tau + dt]$ can be approximated by $q_j dt$ and this indifference becomes:

$$\Pi_i^L e^{-mr_i} = [q_j dt] \Pi_i^F e^{-(m+dt)r_i} + [1 - q_j dt] \Pi_i^L e^{-(m+dt)r_i} \quad (3)$$

The left hand side gives i 's payoff from committing. Since sales take place once both firms have announced price, the leader profit is received when the rival reacts. Hence the profit is

discounted by e^{-mr_i} . The right hand side gives the expected payoff from waiting a short time dt . With probability $q_j dt$, the rival will commit in this interval. The firm would then follow m later, and receive the discounted follower profit. With a probability $(1-q_j dt)$, however the rival will wait, in which case the firm will get the discounted continuation value of the game. Since the equilibrium is stationary and the firm is indifferent between leading and waiting at each point in time, the continuation value is equal to $\Pi_i^L e^{-mr_i}$. Solving (3) for q_j and letting $dt \rightarrow 0$ yields the q_j that makes firm i indifferent between committing and waiting an instant to announce price:

$$q_j = \frac{r_i \Pi_i^L}{\Pi_i^F - \Pi_i^L} \quad \forall i, j \in \{A, B\} \quad i \neq j \quad (4)$$

This is positive and exists so the proposed strategies do form an equilibrium. This mixed-strategy equilibrium is subgame perfect: At any point in time where neither firm has announced price the firms' problem is the same and they will be willing to choose the same strategies.

□

In markets where firms face any cost of delay, however small, leader-follower outcomes can only occur as the result of a mixed-strategy equilibrium. This mixed-strategy equilibrium takes the natural interpretation of a war of attrition. Since each firm prefers the follower position, the disagreement over roles leads to a struggle as each firm tries to gain the more profitable position. Each firm would like to wait to quote price with the hope of getting the rival to announce price early. However, each firm would consider moving early and accepting the leader profit in order to avoid the cost of delay in the event that the rival waits as well.

Notice that firms would face the same incentives if they had existing prices as long as their flow profits from these existing prices were less than the flow profits as a price leader. Suppose that x_i is the present value of firm i 's flow profits over the interval m at the existing prices and redefine Π_i^L and Π_i^F as the present value of the infinite stream of profits as a leader or follower respectively. If sales at the new prices take place once both firms have announced their new prices then both the left and right hand side of equation (3) will go up by x_i , as will the continuation value of the game. If m is small the effect on the continuation value is small and

the other changes cancel out. Thus for small m the basic problem is unchanged: As $m \rightarrow 0$ the resulting equilibrium q_s would approach those found in Claim 2. However, if sales at a new price start taking place the moment the new price is announced, then the firms will have to take into account the fact that their choice of price will affect the profits they receive in the interval m , the amount of time the rival needs to react. These profits, and hence the choice of leader price and probability of leading, will depend on the initial price of the rival. Nevertheless, if m is small the relative importance of these profits while waiting for the rival to react will be small, and the basic nature of the game will be unchanged.

IV. Discussion

In the war of attrition equilibrium differences between the firms translate into asymmetric probabilities of announcing price (equation 4). When firms have identical discount rates, the war of attrition equilibrium implies that the firm with the relatively high follower/leader profit ratio (Π^F/Π^L) will tend to lead more often. While this ratio appears fairly often in the endogenous leadership literature, to date there are no general results on how different factors influence it. In the absence of general results, one can look to the existing literature to see how various economic factors influence the ratio in specific examples.¹⁵

In Deneckere, Kovenock and Lee (1992) the firm with a larger segment of brand loyal consumers emerges as the single price leader. Their results are not directly comparable to ours due to their discontinuous reaction functions and restriction on the timing of moves. Nevertheless, we can examine their profit function pairs for the follower/leader profit ratio. As the firm with the larger segment of loyal costumers increases its customer base further, the relative follower/leader profit ratio of the firm with a smaller base goes down. As the number of switchers decreases – as brand loyalty becomes more important – the follower/leader profit

¹⁵One should be careful in interpretation, however, as many of the examples in the literature which can be solved in closed form involve non-differentiable reaction functions or restrictions on the timing of firms' moves and hence are not directly comparable to the model here.

ratio of the small firm goes down. In our framework in both cases this would imply a higher probability for the larger firm to lead.

Deneckere and Kovenock (1992) examine the effect of capacity constraints on leadership. In their framework too, the follower/leader profit ratio is important. The way the ratio relates to the discount factor yields different predictions on the identity of leader. In contrast to the results in Deneckere, Kovenock and Lee (1992), Deneckere and Kovenock (1992) find that when firms are significantly different from each other the follower/leader profit ratio is higher for the smaller firm. There seems to be room in the literature to further study the determinants of the follower/leader profit ratio and search for more general, unifying results on it.

The war of attrition equilibrium also implies that the firm that is facing a lower cost of delay (a lower r) will be more likely to be the price leader. To the extent that large firms have access to lower interest rates than smaller firms, which is often found empirically,¹⁶ large firms would be more likely to lead price changes. This is the pattern which is observed in many industries.¹⁷

V. Conclusion

This paper analyzes the types of leader-follower outcomes that are likely to arise as the result of non-cooperative competition between similar, symmetrically informed duopolistic firms. In industries where firms face a strict deadline for price announcements and have any cost of delay, however small, all equilibria involve simultaneous price announcements. In the absence of a deadline, however, there exists an equilibrium where both firms mix over the timing of their moves and a leader-follower pattern in pricing may be observed. This mixed-strategy equilibrium takes the interpretation of a war of attrition, since both firms struggle to capture the follower position. If this equilibrium is repeated for successive price changes, the paper predicts

¹⁶See Gertler and Gilchrist (1994) for references to the empirical evidence. See Martinelli (1997) for a theoretical explanation.

¹⁷See Stigler (1947) for evidence from U.S. industries.

occasional changes in the identity of the endogenous price leader. This captures one of the “distinguishing characteristics” of price leadership in industries that do not have a dominant firm (Scherer and Ross, 1990, pp. 249). The probabilities that each firm becomes the endogenous price leader are also derived.

As early as 1934, von Stackelberg argued that in oligopolistic markets the conflict between the firms over the leader and follower roles will lead to an unstable market, as firms continually struggle to gain the advantageous position. “A regular trial of strength emerges and no equilibrium position is reached.... [The pure-strategy “Stackelberg” leader-follower] equilibrium is unstable, for the passive seller can always take up the struggle again at any time.... [E]ach duopolist tries in each case to induce his competitor to adopt another behavior pattern so that he is forced to give in.” (von Stackelberg, 1952, pp.194). Von Stackelberg predicts the identity of the leader firm will vary as a result of this struggle. Likewise, in an influential paper, Markham (1951) argued that “there are certain visible market features associated with competitive price leadership. . . .[I]n the absence of conspiracy one would certainly expect occasional changes in the identity of the price leader” (pp. 897). The mixed-strategy equilibrium presented here lends theoretical support to Markham’s arguments and it can be thought of as a formalization of von Stackelberg’s intuitive claims.

Appendix A

Lemma 1: *Let the probability of committing to a price at time τ if neither firm has yet announced its price be denoted by $\tilde{q}_i \forall i \in \{A, B\}$, then:*

- i) For any $\tilde{q}_i \in [0, 1]$ firm i 's commitment price is unique. Firms will never mix in prices.*
- ii) If $\tilde{q}_j \in (0, 1)$ then firm i 's commitment price is $p_i^C \in (p_i^S, p_i^L)$.*

Proof: So that the proof will apply to both the discrete-time and continuous-time frameworks, let $\delta_i \in (0, 1]$ be the rate at which the profit from having the rival firm react to one's own price is discounted. Thus in the discrete-time framework $\delta \in (0, 1]$, and in the continuous-time framework, $\delta = e^{-\delta t}$. i) Suppose that firm j is playing a mixed-strategy where its commitment price p_j^C has the p.d.f. $f(p_j^C)$ defined on $[0, \infty)$. If firm i chooses to announce its price at τ , its commitment price maximizes its expected profit:

$$\Pi_i = \tilde{q}_j \int_0^\infty f(x) \Pi_i(p_i, x) dx + (1 - \tilde{q}_j) \delta_i \Pi_i(p_i, R_j(p_i)) \quad (5)$$

Notice that Π_i and $\Pi_i(p_i, R_j(p_i))$ are strictly concave in p_i so there is a unique price p_i^C that maximizes the profit of firm i when it commits. By the same argument, there is a unique price p_j^C that maximizes the profit of firm j when it commits to a price. ii) Therefore we can drop the integral and the first-order condition for i 's commitment price is given by:

$$q_j \left[\frac{\partial \Pi_i(p_i, p_j^C)}{\partial p_i} \right] + (1 - q_j) \delta_i \left[\frac{\partial \Pi_i(p_i, R_j(p_i))}{\partial p_i} + \frac{\partial \Pi_i(p_i, R_j(p_i)) \partial R_j}{\partial p_i} \right] = 0 \quad (6)$$

$i, j \in \{A, B\}$ and $i \neq j$. Assume $\tilde{q}_i \in (0, 1)$, if $p_i^C \geq p_i^L$, then the second bracketed term is non-positive, since it is equal to zero at p_i^L . Therefore the first bracketed term must be non-negative, implying that $p_i^C \leq R_i(p_j^C)$. $p_i^F = R_i(p_j^L) < p_i^L \leq p_i^C \leq R_i(p_j^C)$, so $p_i^C \geq p_i^L$ implies $p_j^C > p_j^L$. By the same argument this implies $p_j^C < R_j(p_i^C)$. But, since simultaneous prices are unique and stable, both $p_i^C \leq R_i(p_j^C)$ and $p_j^C < R_j(p_i^C)$ are not possible when $p_i^C > p_i^S$ and $p_j^C > p_j^S$. Hence $p_i^C < p_i^L$.

If $p_i^C \leq p_i^S$, the second bracketed term is positive since $p_i^S < p_i^L$. Therefore the first bracketed term must be negative, implying $p_i^C > R_i(p_j^C)$. $p_i^S = R_i(p_j^S) \geq p_i^C > R_i(p_j^C)$, so $p_i^C \leq p_i^S$ implies $p_j^C < p_j^S$. By the same argument this implies $p_j^C > R_j(p_i^C)$. But, since simultaneous prices are unique and stable, both $p_i^C > R_i(p_j^C)$ and $p_j^C > R_j(p_i^C)$ are not possible when $p_i^C \leq p_i^S$ and $p_j^C \leq p_j^S$. Hence $p_i^C > p_i^S$. \square

Lemma 2: $\Pi_i(R_i(p), R_j(R_i(p))) > \Pi_i(p_i^S, p_j^S) \quad \forall p \in (p_j^S, p_j^L]$ and $i, j \in \{A, B\} \quad i \neq j$.

Proof: $R_i(p) > p_i^S$ since $p > p_j^S$, $p_i^S = R_i(p_j^S)$ and $R_i(\cdot)$ is upward sloping. $R_i(p) \leq p_i^F$ since $p_i^F = R_i(p_j^L)$ and $p \leq p_j^L$. Therefore $R_i(p) < p_i^L$ since $p_i^F < p_i^L$. Since the profit function is increasing in the rival's price and $\Pi_i(p_i, R_j(p_i))$ is concave in p_i , $\Pi_i(p_i, R_j(p_i))$ increases monotonically in the range $[p_i^S, p_i^L)$.

Since $R_i(p) \in (p_i^S, p_i^L)$, $\Pi_i(R_i(p), R_j(R_i(p))) > \Pi_i(p_i^S, p_j^S) \quad \forall p \in (p_j^S, p_j^L]$. □

Lemma 3: *In the continuous-time framework there does not exist a subgame-perfect pure-strategy leader-follower equilibrium. All pure-strategy equilibria involve firms setting simultaneous prices at the same time as, or in the open interval near, each other.*

Proof: Define z_i and z_j as the deterministic price-announcement times that result from the pure strategies s_i and s_j . Without loss of generality, assume that $z_i \leq z_j$. If $z_j > z_i + \epsilon$, then s_j cannot be optimal given s_i since firm j would strictly prefer to make its price announcement at z_i , avoiding the cost of delay. If z_j is in the open interval around z_i then $z_j < z_i + m$ so j 's price cannot be conditional on i 's price. Thus both firms must be on their reaction functions yielding price announcements of $p_i^S = R_i(p_j^S)$ and $p_j^S = R_j(p_i^S)$, an equilibrium which is observationally equivalent to simultaneous price competition. □

Lemma 4: *In the continuous time framework for short reaction times (small m), a deadline and $\tau \leq N - m$, if the equilibrium of every subgame starting at $t > \tau$ results in the deterministic announcement of firms' simultaneous prices, then the equilibrium of the subgame starting at τ results in the deterministic announcement of firms' simultaneous prices.*

Proof: By Lemma 3 all pure-strategy equilibria result in the announcement of firms' simultaneous prices. By Lemma 1 firms will not mix in prices. The only remaining possibility is that there is an equilibrium in mixed-strategies where firms randomize over the timing of their price announcements. Suppose in the subgame starting at τ there existed a non-degenerate mixed-strategy equilibrium where i commits at time τ with the (possibly non-atomistic) probability $\tilde{q}_i \in (0, 1)$. The commitment price of i is denoted by p_i^C . For no $\tilde{q}_i \in (1, 0)$, will firm j be indifferent between committing and waiting. If j waits, with probability \tilde{q}_i , it captures the follower position. With probability $(1 - \tilde{q}_i)$, however, the firms end up in a subgame resulting in announcement of

simultaneous prices. Letting k be the amount of time until both firms will announce their price in that subgame, j 's expected profit from waiting is,

$$\tilde{q}_i e^{-mr_j} \Pi_j(R_j(p_i^C), p_i^C) + (1 - \tilde{q}_i) e^{-kr_j} \Pi_j^S \quad (7)$$

Firm j 's expected profit from quoting $R_j(p_i^C)$ at τ is,

$$\tilde{q}_i \Pi_j(R_j(p_i^C), p_i^C) + (1 - \tilde{q}_i) e^{-mr_j} \Pi_j(R_j(p_i^C), R_i(R_j(p_i^C))) \quad (8)$$

for small m (short reaction times) (8) > (7) by Lemmas 1 and 2. Firm j would always strictly prefer committing to waiting, since j can avoid any chance of the low simultaneous profit by committing to the price $R_j(p_i^C)$ at time τ . Hence there does not exist a $q_i \in (0, 1)$ that would make j indifferent between announcing its price at τ and waiting: Firm j must be playing a pure strategy in the subgame beginning at τ . If j is moving deterministically then i cannot be mixing either, and the equilibrium must be in pure strategies.

□

Appendix B

This appendix shows that in the continuous time model with a deadline for price announcements there does not exist a subgame-perfect mixed-strategy equilibrium. Let date N be the deadline for price announcements. In equilibrium firms cannot be mixing in prices by Lemma 1. Consider then the possibility that they might be mixing over the date of their price announcements.

In the range $t \in (N-m, N]$ if neither firm has announced its price by time t , then there is no time left for either firm to react to its rival's price, so both firms must be on their reaction functions resulting in simultaneous prices. Given this, there cannot be a mixed-strategy equilibrium in the subgame starting at t . If firm i was going to mix over its announcement time then its rival would strictly prefer not to delay its announcement past the first date with a positive probability of a price announcement. Early price announcement would avoid any chance of incurring a cost of delay. In this case, however, firm i would also not wish to delay past that date. Thus all equilibria in all subgames in the range $t \in (N-m, N]$ result in deterministic announcement of simultaneous prices.

Lemma 4 shows that if the equilibrium of every subgame starting at $t > \tau$ involves deterministic announcement of simultaneous prices, then the equilibrium of the subgame starting at τ also involves deterministic announcement of simultaneous prices. Thus if neither firm has announced its price by time $N-m$ then firms will deterministically announce their simultaneous in the subgame starting at time $N-m$. From this, Lemma 3 can be applied to the open interval before $N-m$ to show that firms must be announcing simultaneous prices if they get there as well. Backward induction via the same argument eliminates mixed strategies at any date.

□

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